SIGNAL PROCESSING II: Theories and Applications
H.W. Schüssler (editor)
Elsevier Science Publishers B.V. (North-Holland)

MATHEMATICAL MORPHOLOGY IN GREY-IMAGES

Karsten Rodenacker, Peter Gais, Uta Jütting, Georg Burger

Gesellschaft für Strahlen- und Umweltforschung mbH München
Institut für Strahlenschutz
D-8042 Neuherberg bei München

The application of methods of mathematical morphology for the analysis of grey-images is shown. The corresponding algorithms are based upon operations used in set-theory and are extended to scalar fields, as represented by grey-images. Derived operations and some algorithms, describing the opening and closing, grassfire and a thinning procedure, leading to a socalled ricefield transformation of structured images, are described and the results of their application upon human cell images are demonstrated.

1. INTRODUCTION

Mathematical morphology is a set-theoretical approach to formalize methods applied on sets of the n-dimensional euclidean space and to obtain quantitative data on their shape and arrangement. It is developed at the school of Fontainebleau (Matheron /1/, Serra /2/). The mathematical morphology is mainly applied on sets in the two-dimensional space. A subset of the plane mapped by its characteristic function inside of a bounded area defines a binary image. This may represent a certain physical property of an object such as, for example, its area under an arbitrary projection. There are some basic operations defined on sets or binary images, respectively, which allow their processing. In chapter 2, we will give a brief introduction of these operations.
In contrast to a binary image, a grey-image is defined by a scalar field, where the scalar may represent again a certain physical property of an object such as, for example, its brightness. A suitable discrimination applied on the grey-values delivers a binary image. The most used discrimination is the threshold at a certain grey value. This necessitates a-priori information on the image which is often not available. To avoid thresholding the operations defined by mathematical morphology on sets or binary images, respectively, were extended to operations on fuzzy sets (Zadeh/3/) or grey images, respectively. This has been done by Serra et al. /2/, Sternberg/4/ and Goetcherian/7/. The primary goal was to define operations on grey-images which will show the same results as the application of the original operations on binary images, generated by thresholding at any possible grey value. Another approach was carried out by Sternberg/4/, who considers sets in the three-dimensional space.
In our group we are mainly involved with applied image analysis on microscopical objects. This includes the completely automated measurement of scenes, their segmentation, evaluation of features and classification of objects. Images are taken by a TV-camera, digitized and read directly into our attached array-processor (AP), on which basic operations of mathematical morphology on binary images as well as on grey-images are implemented. In chapter 4 the application of some of the operations is shown with respect to scene - as well as object-analysis under the notions

- cleaning of scenes
- restorations of objects
- separation of objects out of scenes
- special transformations with respect to local properties.

2. THEORY

The basic operations on sets and on binary images are as follows:

	SETS	BINARY IMAGES
Union	$X \cup Y$	X.OR.Y
Intersection	$X \cap Y$	X.AND.Y
Complementation	X^c	.NOT.X
Difference	$X \setminus Y$	X.AND..NOT.Y
Translation by a vector a	$X + a = \{X + a \mid x \in X\}$	.SHIFT.(X,a)

From these the following operations of mathematical morphology are deduced:

Erosion $\quad X \ominus B = \bigcap \{X + b \mid b \in B\}$

Dilation $\quad X \oplus B = \bigcup \{X + b \mid b \in B\}$

B and C are the sets of vectors for translation. They are called structuring elements.

Hit-or-miss transformation $\quad X \circledast (B,C) = (X \oplus B) \cap (X \ominus C)^c$

Opening $\quad X_B = (X \ominus B) \oplus B$

Closing $\quad X^B = (X \oplus B) \ominus B$

For grey-images the basic operations have to be redefined. This is only possibly under certain conditions (Serra/2/). In our approach it is sufficient that grey values are non-negative and bounded. That is always given for real images.

Union $X \cup Y = \max(X,Y)$

Intersection $X \cap Y = \min(X,Y)$

Complementation $X^c = m - X$
(Arithmetical complement)

Difference $X \setminus Y = X - Y$
(Subtraction, defined only if $Y \leq X$)

Translation no change

All operations have to be performed elementwise. Complementation and difference are not directly comparable with the corresponding operators in binary images, since the grey-images build no boolean algebra in contrast to the binary images. Based upon this, all deduced operations, as defined for binary images, except the hit-or-miss transformation, are now applicable on grey-images. Instead of the hit-or-miss transformation a thinning algorithm (Serra/2/) was implemented as follows:

Let X(z) be the grey value at the location z than

$$X \circledast (B,C)(z) = X \ominus C(z) \quad \text{if} \quad X \oplus B(z) > X(z) \geq X \ominus C(z)$$

$$= X(z) \quad \text{else}$$

With $B = 0$ is $X \circledast (0,C) = X \ominus C$
and with $C = 0$ is $X \circledast (B,0) = X \oplus B$.

So this operation is of multifunctional purpose and can be used quite arbitrarily.

3. IMPLEMENTATION

The images are digitized on a rectangular grid. Erosion and dilation are performed with an approximately spherical structuring element centred around the zeropoint of the plane with arbitrary radius. An octogon or a sixteen sided polygon can be chosen. The hit-or-miss transformation is implemented for structuring elements with sizes up to 5 x 5 pixels centred around the zeropoint of the plane. With the latter a skeleton (Meyer /5/) on binary images and a watershed (Goetcherian/7/) on grey-images is performed which allows us to continue our work especially on the segmentation and measurement of texture in cell nuclei (Rodenacker et al./6/). All functions are programmed in a so called Vector-Function-Chainer Language which allows one to program the AP a bit more comfortably than in assembler.

4. APPLICATION

The automatical processing of microscopic images includes digitization, shading correction, segmentation into objects (as cells) and background, deletion of undesired objects like artefacts, segmentation of single objects into different parts (as cytoplasm and nucleus) but also the further subdivision of nuclei into structural parts of the chromatine. At the end of this process, object specific features are determined. In the following we will show the application of mathematical morphology on grey-images according to the same sequence of processing. The first important step is object segmentation. To prepare the scene for improved segmentation and to remove small non-interesting objects an image clearance with following restorations is performed.

Image clearance:

An opening algorithm in the grey-image (fig. 1) with suffiently large approximately circular shaped structuring element B1 removes all small objects, into which the element B1 does not fit completely. The result X_{B1} is shown in figure 2. The removal of too large objects is done later in the segmented binary image, again by means of an opening algorithm. The image X_{B1} shows some object modifications according to the structuring element B1, necessitating image reconstruction.

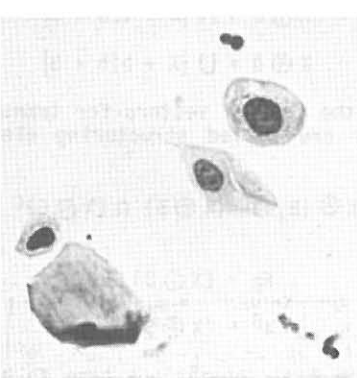


Figure 1 : Scene with different objects

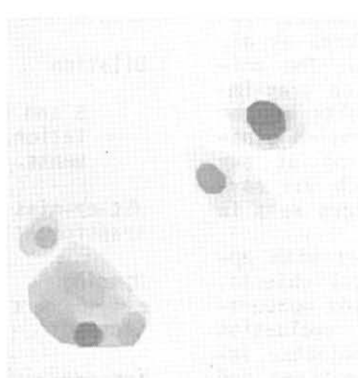


Figure 2 : Opening of fig.1

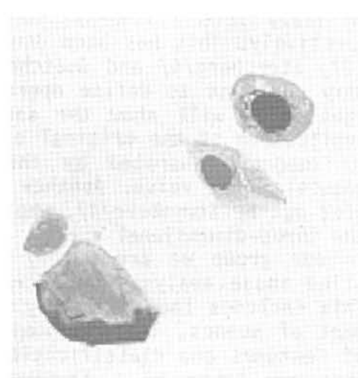


Figure 3 : Grassfire on fig.2 with fig.1

Image restoration:

An optimal restoration of object shapes, however, not of the internal substructure of the objects is performed by the following grassfire algorithm applied on the 'label-image' X_{B1}.

```
procedure grsf (Z,X_B1,X,C,e);
                    (* grassfire algorithm *)
    begin
    Y_0 = 0; Y_1 = X_B1;
    while d(Y_0,Y_1) > e  do
        begin
        Y_0 = Y_1; Y_1 = (Y_1 ⊕ C) ∩ X
        end;
    Z = Y_1;
    end;
```

The result Z represents the reconstructed image (fig.3), $d(Y_0,Y_1)$ is a distance function on grey-images (Guedj/8/), C is a tiny structuring element which prevents reconstruction of unlabelled objects. The grassfire burns, that means the dilation of the labelled objects, as long as the grey values of the dilated regions are higher than those of the underlying original image.

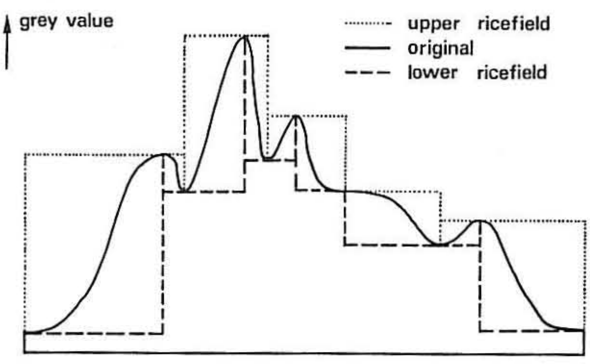


Figure 4 : Schematic result of the thinning algorithm (1-dim)

Object separation:

To separate single objects from each other, a grassfire algorithm can be started at the pixel with highest grey value, occurring for example in the image Z (fig.3). After restoration the selected object will be quenched, and the procedure repeated. This leads to sequential object identification and ranking according to the maximal point occurring in them. The separated objects can now be easily segmented by a threshold algorithm.

'Thinning' Transformation:

As already mentioned the grassfire image restoration taking into account only monotoneous decreasing grey values destroys a good deal of the internal structure of objects. To display such structures formed for example by the chromatine in cell nuclei, an algorithm was implemented, describing the local arrangement of high and low grey values in terms of watershed contours and different altitude level regions defined by them (fig.4). The algorithm is as follows:

```
procedure rcf (Z,X,e);
                    (* ricefield transformation *)
    begin
    Y_0 = 0; Y_1 = X;
    while d(Y_0,Y_1) > e  do
        begin
        Y_0 = Y_1;
        for i = 1  to  8  do
            Y_1 = Y_1 ⊛ (B^i,C^i);
        end;
    Z = Y_1;
    end;
rcf (L,X,e);            (* lower ricefield *)
rcf (U,X^C,e);          (* upper ricefield *)
```

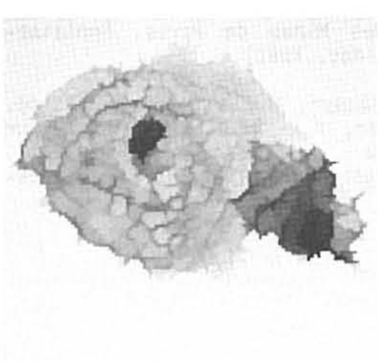


Figure 7 : Lower ricefield

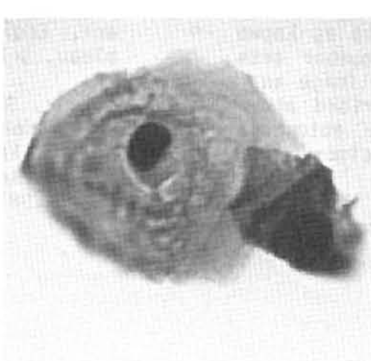


Figure 6 : Original cell image

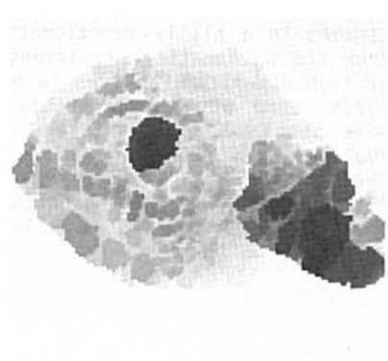
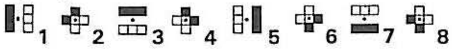


Figure 8 : Upper ricefield

The function $d(Y_0,Y_1)$ again describes a so-called distance on grey-images. B^i (■) and C^i (□) are structuring elements of different shapes, as shown below:



The transformation results in watershed contours in the original as well as in the complemented image, the latter displaying sink or valley-bed lines in the original.

If one defines the valley regions in between the watersheds and the scene boundaries as flat terraces with the altitude of the lowest point in the valleys one gets the picture of 'ricefield terraces' (fig. 5). These terraces of the

Figure 5 : Ricefield terraces

original and the complemented image are called lower ricefield regions L, and upper ricefield regions U. The difference of both ricefield representations $D = U \setminus L$ describes in a characteristic manner the topology of the grey-image. The following figures 6,7,8 demonstrate for an epithelial cell with an adhering artefact the results of the described transformations (Burger et al./9/).

5. CONCLUSION

Mathematical morphology on grey-images is a powerful tool for the processing of images without a-priori information. It allows to manipulate grey-images in theoretical and practical manners in a highly functional degree as known from the mathematical treatment of number sets or from algorithms applied in binary image analysis. Some of the algorithms described above have been used successfully in the automatic evaluation of cellular aspirate specimens from the thyroid.
On the other side the handling of the corresponding algorithms is not always easy due to the lack of possible definitions and descriptions of typical image elements as objects, edges, borders, etc. A grey-image always covers the whole area, whereelse discrimination into objects and subregions of interest is normally performed in segmented binary images.
Additionally the effects of mathematical morphology operations on grey-images are often difficult to understand in terms of visual perception, especially when the original grey-image itself is a transformed one as, for example a raster scanned electron microscopic image.

ACKNOWLEDGEMENT

We acknowledge many helpful discussions with Dr. F. Meyer, Fontainebleau, as well as the fruitful collaboration with M. Guedj.

REFERENCES

/1/ Matheron, G., Random Sets and Integral Geometry (Wiley, New York, 1975)

/2/ Serra, J., Image Analysis and Mathematical Morphology (Academic Press, London, 1982)

/3/ Zadeh, L.A., Fuzzy Sets, Information and Control 8 (1965) 338 - 353

/4/ Sternberg, S.R., Cellular Computers and Biomedical Image Processing, in Sklansky, J. and Bisconte, J.-C. (eds.), Biomedical Images and Computers (Springer Verlag, Berlin, 1982) 294 - 319

/5/ Meyer, F., Cytologie Quantitative et Morphologie Mathématique, Thèse, Ecole des Mines de Paris (1979)

/6/ Rodenacker, K., Gais, P., Jütting, U., Segmentation and Measurement of the Texture in Digitized Images, Stereol. Jugosl. 3 Supp.1 (1981) 165 - 174

/7/ Goetcherian, V., From Binary to Grey Tone Image Processing Using Fuzzy Logic Concepts, Pattern Recognition 12 (1980) 7 - 15

/8/ Guedj, M., Rapport sur le stage d'ingénieur, Ecole des Mines de Paris, Fontainebleau, S-128 (Nov. 1982)

/9/ Burger, G., Abmayr, W., Gais, P., Jütting, U., Rodenacker, K., Bildanalysesystem zur Klassifikation zytologischer Präparate, BMFT-Forschungsbericht T 81-148 FIZ, Karlsruhe (1981)